

Ghost Condensation in the Brane-World

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Abstract

Motivated by the ghost condensate model, we study the Randall-Sundrum (RS) brane-world with an arbitrary function of the higher derivative kinetic terms, $\mathcal{P}(X)$, where $X = -(\nabla\phi)^2$. The five-dimensional Einstein equations reduce to two equations of motion with a constraint between $\mathcal{P}(X)$ and the five-dimensional cosmological constant on the brane. For a static extra dimension, $\mathcal{P}(X)$ has solutions for both a negative kinetic scalar (so called *ghost*) as well as an ordinary scalar field. However ghost condensation cannot take place. We show that small perturbations along the extra dimensional radius (the radion) can give rise to ghost condensation. This produces a radiation-dominated universe and the vanishing cosmological constant at late times but destabilizes the radion. This instability can be resolved by an inclusion of bulk matter along y -direction, which finally presents a possible explanation of the late-time cosmic acceleration.

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1 Introduction

Physics in spacetimes with extra dimensions yields many theoretical insights and poses new experimental challenges. Though the idea of introducing higher dimensions has been around for more than 80 years (since the Kaluza-Klein proposal), it has been successfully reinvented since the advent of string theory. In large part this is due to the general expectation that the existence of additional spatial coordinates might resolve a number of problematic issues such as the smallness of cosmological constant, the hierarchy between energy scales, and the accelerating expansion of the universe, that consideration becomes of great interest and importance in the last few years.

From this perspective the brane-world scenario offers tantalizing new prospects for addressing puzzling issues rooted in both cosmology and particle physics. Pioneering work by Randall and Sundrum (RS) [1] posited two models with non-flat extra dimensions in which the universe is regarded as a three-dimensional brane located at a fixed point of an \mathbf{S}^1/Z_2 orbifold in five dimensions. The zero modes of the gravitational field, which turn out to be massless on the branes, can be trapped on the brane for perturbations around a flat brane geometry. In the context of string theory, the context of the model arises from the $E_8 \times E_8$ heterotic string theory related to an eleven-dimensional supergravity theory on orbifold $\mathbf{R}^{10} \times \mathbf{S}^1/Z_2$ [2]. The RS brane-world scenario has drawn much attention because it offers new possibilities for addressing both the gauge hierarchy problem and the cosmological constant problem. Its rather different cosmological perspective generated several attempts to recover conventional cosmology from it. These include radion stabilization mechanisms [3, 4, 5, 6, 7, 8, 9] and related works associated with inflation [10] and quintessential brane models [11].

On the other hand, Dvali *et al.* suggested a brane-induced model with a flat large extra dimension in five dimensions [12], which shows that the theory on a 3-brane in five dimensional Minkowski spacetimes gives rise to the four-dimensional Newtonian potential at short distances while the potential at large distances is that of the five-dimensional theory. Therefore, the model was regarded as an interesting attempt at modifying gravity in the infrared (IR) region. In the context of IR modification of general relativity, another intriguing suggestion is the *ghost condensation* mechanism [13] in which a condensing ghost field forms a sort of fluid with an equation-of-state parameter, $\omega = -1$, where $\omega = p/\rho$, that fills the universe but has different properties from that of a cosmological constant. The ghost condensate breaks time-translation symmetry (a kind of Lorentz symmetry breaking), making the graviton massive in the IR region and giving rise to a stable vacuum state in spite of a wrong-signed kinetic term. This is somewhat like a gravitational analogue of the

Higgs mechanism, and the model has been proposed as a candidate that could account for a consistent IR modification of general relativity and for a connection between inflation and the dark energy/matter [14, 15, 16].

Although incorporating ghosts in a cosmological model with extra dimensions has been suggested before [17, 18], a model with condensing ghosts offers a mechanism for producing matter or vacuum energy from exotic objects that has not yet been considered. Motivated by the preceding considerations, in this paper we explore the consequences of combining the brane-world scenario with the ghost condensate model. We find that ghost condensation is not possible with a static extra dimension. A dynamic radion field can provide a mechanism for condensation, but is generically unstable. Another way to achieve ghost condensation is by introducing bulk matter. This can make the radion static, but has the effect of reducing the system to the four-dimensional case studied in ref.[16].

In Sec. 2, we present the five-dimensional Einstein-Hilbert action with the cosmological constant, brane tensions, and an arbitrary function of the scalar kinetic terms, $\mathcal{P}(X)$. The field equation for $\mathcal{P}(X)$ is investigated. In Sec. 3, the brane junction conditions are imposed and the five-dimensional equations of motion with a constraint are derived for a static extra dimension. The Friedmann and the acceleration equations on the brane are derived by following the Binétruy-Deffayet-Langlois (BDL) type approach [3, 7, 8, 9]. We show that the conservation equations on the brane are satisfied. In Sec. 4, we show that small perturbations of the brane along the extra dimension can give rise to ghost condensation and the equations at late times can determine the vacuum as Minkowski spacetime. Although the model describes the radiation-dominated universe at late times, it is accompanied by an unstable radion field. We address this issue in Sec. 5, where we show that the instability of the radion field can be resolved by introducing bulk matter along the y -direction. This bulk matter can make the radion stable, and reduces the system to that of the four-dimensional case, which has an inflationary solution for the scale factor and a late-time cosmic speed-up. This also offers consistent solutions with the stable radius of extra dimension, describing a de-Sitter (dS) phase in which the resulting evolution of the ghost condensate behaves like a cosmological constant with $\omega = -1$ at late times. Finally, some discussion and comments on our results are given in Sec. 7.

2 Setup and Equations of Motion

Let us consider a five-dimensional spacetime with a \mathbf{S}^1/Z_2 orbifold structure along the fifth direction. The five-dimensional Einstein-Hilbert action with the matter lagrangian is given

by

$$S = \int d^4x dy \sqrt{-\hat{g}} \left[\frac{M_{(5)}^4}{2} \hat{R} + \hat{\mathcal{L}}_M \right], \quad (2.1)$$

where hatted quantities are five-dimensional and $M_{(5)}^4 = 1/8\pi G_{(5)}$, with $G_{(5)}$ the five-dimensional gravitational constant. The most general cosmological metric ansatz consistent with the orbifold structure is given by the following line element

$$(ds)^2 = -n^2(t, y) dt^2 + a^2(t, y) h_{ij} dx^i dx^j + b^2(t, y) dy^2, \quad (2.2)$$

where h_{ij} is a 3-dimensional homogeneous and isotropic induced metric defined by

$$h_{ij} = \delta_{ij} + \frac{\alpha x_i x_j}{1 - \alpha x^m x^n \delta_{mn}}, \quad (2.3)$$

where α represents its spatial curvature which has the value of $0, \pm 1$.

From the metric (2.2), we obtain

$$\hat{G}_{00} = 3 \left[\left(\frac{\dot{a}}{a} \right)^2 + \frac{\dot{a}\dot{b}}{ab} - \frac{n^2}{b^2} \left(\frac{a''}{a} - \frac{a'b'}{ab} + \left(\frac{a'}{a} \right)^2 \right) + \frac{n^2}{a^2} \alpha \right], \quad (2.4)$$

$$\begin{aligned} \hat{G}_{ij} = & -h_{ij} \frac{a^2}{n^2} \left[\left(\frac{\dot{a}}{a} \right)^2 + 2 \frac{\ddot{a}}{a} - 2 \frac{\dot{n}\dot{a}}{na} + 2 \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{n}\dot{b}}{nb} + \frac{\ddot{b}}{b} \right] \\ & + h_{ij} \frac{a^2}{b^2} \left[\left(\frac{a'}{a} \right)^2 + 2 \frac{a''}{a} - 2 \frac{a'b'}{ab} + 2 \frac{n'a'}{na} - \frac{n'b'}{nb} + \frac{n''}{n} \right] - \alpha h_{ij}, \end{aligned} \quad (2.5)$$

$$\hat{G}_{44} = 3 \left[\left(\frac{a'}{a} \right)^2 + \frac{a'n'}{an} - \frac{b^2}{a^2} \alpha - \frac{b^2}{n^2} \left(\left(\frac{\dot{a}}{a} \right)^2 + \frac{\ddot{a}}{a} - \frac{\dot{n}\dot{a}}{na} \right) \right], \quad (2.6)$$

$$\hat{G}_{04} = 3 \left[\frac{n'\dot{a}}{na} - \frac{\dot{a}'}{a} + \frac{a'\dot{b}}{ab} \right], \quad (2.7)$$

for the Einstein tensor, where the dot and the prime respectively denote t and y derivatives.

The matter lagrangian $\hat{\mathcal{L}}_M$ includes the brane tension, $\hat{\mathcal{L}}_M^{\text{brane}}$, the five-dimensional cosmological constant, $\hat{\mathcal{L}}_M^{\Lambda(5)}$, the bulk matter along the y -direction, $\hat{\mathcal{L}}_M^{\text{bulk}}$, and an arbitrary function of the kinetic terms, $\hat{\mathcal{L}}_M^{\mathcal{P}}$,

$$\hat{\mathcal{L}}_M = \hat{\mathcal{L}}_M^{\text{brane}} + \hat{\mathcal{L}}_M^{\Lambda(5)} + \hat{\mathcal{L}}_M^{\text{bulk}} + \hat{\mathcal{L}}_M^{\mathcal{P}} \quad (2.8)$$

where the lagrangians of the brane tension and cosmological constant yield in turn simple expressions for their energy-momentum

$$\begin{aligned} \hat{T}_N^{M \Lambda(5)} + \hat{T}_N^{M \text{bulk}} + \hat{T}_N^{M \text{brane}} = & \text{diag}(-\rho_\Lambda, p_\Lambda, p_\Lambda, p_\Lambda, p_\Lambda) \\ & + M_{(5)}^4 \text{diag}(0, 0, 0, 0, p_5) + \sum_{u=0, \pi} \frac{\delta(y_u)}{b} \text{diag}(-\rho_u, p_u, p_u, p_u, 0) \end{aligned} \quad (2.9)$$

where u represents the visible ($u = 0$) and hidden ($u = \pi$) branes [1], and we have $\rho_\Lambda = -p_\Lambda = \Lambda_{(5)}$, where $\Lambda_{(5)} < 0$. In addition, p_5 is bulk matter along the y -direction that is responsible for the radion stabilization [7, 17]. Note that the indices M, N, \dots and μ, ν, \dots run from 0 to 4 and 0 to 3, respectively. And we denote $i, j, \dots = 1, 2, 3$, which represents the spatial coordinates on the three-dimensional brane.

On the other hand $\hat{\mathcal{L}}_M^{\mathcal{P}}$, the lagrangian for $\mathcal{P}(X)$, can be written as

$$\hat{\mathcal{L}}_M^{\mathcal{P}} = M_{(5)}^4 \mathcal{P}(X), \quad (2.10)$$

where $\mathcal{P}(X)$ is an arbitrary function of $X = -\partial_M \phi \partial^M \phi$ that describes the higher derivative terms of the kinetic scalar field. This type of matter was considered in connection with k-essence [19] and ghost condensation [13]. The energy-momentum tensor of eq. (2.10) is

$$\hat{T}_{MN}^{\mathcal{P}} = M_{(5)}^4 \left(\mathcal{P}(X) \hat{g}_{MN} + 2 \frac{\partial \mathcal{P}(X)}{\partial X} \partial_M \phi \partial_N \phi \right). \quad (2.11)$$

Einstein's equation is easily obtained from eqs. (2.1), (2.9), and (2.11),

$$\hat{G}_{MN} = \frac{1}{M_{(5)}^4} \hat{T}_{MN} = \frac{1}{M_{(5)}^4} (\hat{T}_{MN}^{\Lambda_{(5)}} + \hat{T}_{MN}^{\text{bulk}} + \hat{T}_{MN}^{\text{brane}} + \hat{T}_{MN}^{\mathcal{P}}). \quad (2.12)$$

The field $\mathcal{P}(X)$ satisfies

$$\partial_M \left(\sqrt{-\hat{g}} \frac{\partial \mathcal{P}(X)}{\partial X} \partial^M \phi \right) = 0, \quad (2.13)$$

which has the solution

$$\frac{a^3 b}{n} \partial_X \mathcal{P}(\dot{\phi}^2) \dot{\phi} = \text{const}, \quad (2.14)$$

for $\phi = \phi(t)$ when $\alpha = 0$. Writing $a^2(t, y) = n^2(t, y) a_0^2(t)$ and $n^2(t, y) = e^{-2b(t, y)\sigma(y)}$ (reflecting the *warped geometry*), the metric (2.2) becomes

$$(ds)^2 = e^{-2b\sigma(y)} \left(-dt^2 + a_0^2(t) h_{ij} dx^i dx^j \right) + b^2(t, y) dy^2, \quad (2.15)$$

and eq. (2.14) can be written as

$$b e^{-2b\sigma} \partial_X \mathcal{P}(\dot{\phi}^2) \dot{\phi} = \frac{\text{const}}{a_0^3}. \quad (2.16)$$

Since we expect that the scale factor a_0 on the brane goes to infinity at late times, at least one of the following three scenarios

$$\dot{\phi} \rightarrow 0 \quad \text{or} \quad \partial_X \mathcal{P}(\dot{\phi}^2) \rightarrow 0 \quad \text{or} \quad b e^{-2b} \rightarrow 0 \quad (2.17)$$

must ensue. The first option is the case that the scalar velocity goes to zero whereas the second leads to ghost condensation at late times; both of these options have been previously considered [13, 16].

The third option arises from the five-dimensional brane-world hypothesis. This case implies that the time-evolution of the extra dimension, $b(t, y)$, has solutions $b = 0$ or $b = \infty$ at late times. Either of these solutions automatically satisfies the field equation at late times. To address the gauge hierarchy problem, the RS1 model assumes a very small extra dimension, congruent with $b \rightarrow 0$, while the $b \rightarrow \infty$ limit is congruent with the RS2 model, which has a semi-infinite extra dimension [1].

In the brane-world scenario, the position of a brane can be stabilized by the radion stabilization mechanism [4]. Once the radius of the extra dimension is fixed, it has a static configuration, $\dot{b} = 0$. We shall explore this option in the next section.

3 Ghosts and a Static Extra Dimension

3.1 Brane Junction Condition and the Ghost

In this section, we consider a model with a static extra dimension and $p_5 = 0$. Since there exist nontrivial topological defects such as 3-branes orthogonal to the fifth direction, an appropriate junction condition is required to resolve the discontinuity. These conditions can be imposed by integrating (00)- and (11)-components of the Einstein's equation, yielding

$$[a'] = -\frac{a_u b_u}{3M_{(5)}^4} \rho_u, \quad (3.1)$$

$$[n'] = \frac{n_u b_u}{3M_{(5)}^4} (2\rho_u + 3p_u) \quad (3.2)$$

on the branes at $y = 0$ and $y = \pi$, where the prime denotes a derivative with respect to y and $[a'] = a'(y_{+u}) - a'(y_{-u})$. Using the junction condition on the branes at $y = 0$, the static extra dimension ($\dot{b} = 0$), and the gauge fixing of $b = r_c = \text{const.}$ and $n_0 \equiv n(t, 0) = 1$, the evolution equations on the brane become

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{1}{3} (2X \partial_X \mathcal{P}(X) - \mathcal{P}(X)) + \frac{\Lambda_{(5)}}{3M_{(5)}^4} + \frac{\rho_0^2}{36M_{(5)}^8}, \quad (3.3)$$

$$H_0^2 + \frac{\alpha}{a_0^2} + 2\frac{\ddot{a}_0}{a_0} = -\mathcal{P}(X) + \frac{\Lambda_{(5)}}{M_{(5)}^4} + \frac{\rho_0^2}{36M_{(5)}^8} - \frac{2\rho_0}{36M_{(5)}^8} (2\rho_0 + 3p_0), \quad (3.4)$$

$$H_0^2 + \frac{\alpha}{a_0^2} + \frac{\ddot{a}_0}{a_0} = -\frac{1}{3} \mathcal{P}(X) + \frac{\Lambda_{(5)}}{3M_{(5)}^4} + \frac{\rho_0^2}{36M_{(5)}^8} - \frac{\rho_0}{36M_{(5)}^8} (2\rho_0 + 3p_0), \quad (3.5)$$

where $H_0^2 = \dot{a}_0^2/a_0^2$ is the Hubble's parameter defined on the brane at $y = 0$. Combining the above three equations yields a simple relation between the function $\mathcal{P}(X)$ and the cosmological constant,

$$\mathcal{P}(X) - X\partial_X\mathcal{P}(X) = \frac{\Lambda_{(5)}}{M_{(5)}^4}, \quad (3.6)$$

which has the general solution

$$\mathcal{P}(X) = \frac{\Lambda_{(5)}}{M_{(5)}^4} + CX, \quad (3.7)$$

where C is an integration constant. Note that if C is positive, this describes an ordinary scalar field (stiff matter) while negative C represents a ghost scalar field. Since there is no restriction on the choice of sign for C , these two solutions are both allowed. Equation (3.7) reduces the Friedmann and acceleration equations to

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{1}{3}CX + \frac{\rho_0^2}{36M_{(5)}^8}, \quad (3.8)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{2}{3}CX - \frac{\rho_0}{36M_{(5)}^8}(2\rho_0 + 3p_0). \quad (3.9)$$

We see from (3.7) that the brane-world does not allow higher derivative kinetic terms if the extra dimension is static. We shall see in sec. 4 that removing this constraint (i.e. assuming a non-static extra dimension) gives rise to ghost condensation.

3.2 Ghosts on the Brane

In this section, we shall derive the Friedmann and the acceleration equations on the brane at $y = 0$ by imposing the brane junction condition using the BDL approach [3, 7, 8, 9]. To investigate the Friedmann equation on the brane at $y = 0$, we define

$$J(t, y) \equiv \frac{(a'a)^2}{b^2} - \frac{(\dot{a}a)^2}{n^2} - \alpha a^2. \quad (3.10)$$

The (00)-and (44)-components of the Einstein's equation in the bulk are then

$$J'(t, y) = \frac{1}{6}\partial_y(a^4)\frac{\hat{T}_t^t(t, y)}{M_{(5)}^4}, \quad (3.11)$$

$$\dot{J}(t, y) = \frac{1}{6}\partial_t(a^4)\frac{\hat{T}_y^y(t, y)}{M_{(5)}^4}, \quad (3.12)$$

where $\hat{T}_N^M = \hat{T}_N^M{}^{\Lambda_{(5)}} + \hat{T}_N^M{}^{\mathcal{P}}$ in the bulk. It is easy to show that above equations satisfy the continuity condition, $\partial_t\partial_y J(t, y) = \partial_y\partial_t J(t, y)$, by using the (04)-component of Einstein's

equation and the conservation of the energy-momentum tensor (or field equation for $\mathcal{P}(X)$). Notice that eqs. (3.11) and (3.12) can be simply solved as

$$J(t, y) = \frac{a^4}{6M_{(5)}^4} \hat{T}_t^t - \frac{1}{6M_{(5)}^4} \int dy a^4 \partial_y \hat{T}_t^t{}^{\mathcal{P}} + \Omega(t) \quad (3.13)$$

$$= \frac{a^4}{6M_{(5)}^4} \hat{T}_y^y - \frac{1}{6M_{(5)}^4} \int dt a^4 \partial_t \hat{T}_y^y{}^{\mathcal{P}} + \tilde{\Omega}(y), \quad (3.14)$$

where $\Omega(t)$ and $\tilde{\Omega}(y)$ are integration functions that respect the relation

$$\Omega(t) = \frac{a^4}{6M_{(5)}^4} (\hat{T}_y^y{}^{\mathcal{P}} - \hat{T}_t^t{}^{\mathcal{P}}) + \frac{1}{6M_{(5)}^4} \left(\int dy a^4 \partial_y \hat{T}_t^t{}^{\mathcal{P}} - \int dt a^4 \partial_t \hat{T}_y^y{}^{\mathcal{P}} \right) + \tilde{\Omega}(y). \quad (3.15)$$

On the other hand, the (04)-component of the Einstein's equation is

$$\frac{n' \dot{a}}{na} - \frac{\dot{a}'}{a} = 0, \quad (3.16)$$

for a static radius of extra dimension, $\dot{b} = 0$, which implies that $\dot{a}(t, y) = \nu(t)n(t, y)$.

From eq. (3.13), we have

$$\frac{a'^2}{b^2} = \nu^2(t) + \alpha - k^2 a^2 + \frac{a^2}{6M_{(5)}^4} \hat{T}_t^t{}^{\mathcal{P}} - \frac{1}{6M_{(5)}^4 a^2} \int dy a^4 \partial_y \hat{T}_t^t{}^{\mathcal{P}} + \frac{\Omega(t)}{a^2}, \quad (3.17)$$

where $k^2 \equiv \Lambda_{(5)}/6M_{(5)}^4$. If we define $\hat{\mathcal{G}}^{\mathcal{P}}(t, y) \equiv \hat{T}_t^t{}^{\mathcal{P}}/6M_{(5)}^4$, then eq. (3.17) becomes

$$\int b k dy = \int \frac{da}{\sqrt{\frac{\nu^2 + \alpha}{k^2} - a^2 + \frac{a^2 \hat{\mathcal{G}}^{\mathcal{P}}}{k^2} + \frac{\Omega}{k^2 a^2}}} \left(1 - \frac{\frac{1}{a^4 k^2} \int dy a^4 \partial_y \hat{\mathcal{G}}^{\mathcal{P}}}{\frac{\nu^2 + \alpha}{k^2} - a^2 + \frac{a^2 \hat{\mathcal{G}}^{\mathcal{P}}}{k^2} + \frac{\Omega}{k^2 a^2}} \right)^{-1/2}. \quad (3.18)$$

Note that the RHS of eq. (3.18) is the integration with respect to only a . However, because of the integration term in the square-root, we cannot get an exact solution of eq. (3.18) even if $\hat{\mathcal{G}}^{\mathcal{P}}$ does not depend upon a . However, at this stage, if we take a near-brane limit $y \rightarrow 0$, then $\partial_y \hat{\mathcal{G}}^{\mathcal{P}}$ vanishes since $n(t, y)|_{y \rightarrow 0} \rightarrow n(t, 0) = n_0(t) = 1$ by gauge fixing. Therefore, we can approximately use the usual BDL approach in this sense.

Integrating eq. (3.18) leads to the following solution for the scale factor

$$a^2(t, y) = \frac{W(t) e^{2\sqrt{\mathcal{F}} b k y}}{2\sqrt{\mathcal{F}}} - \frac{\nu^2(t) + \alpha}{2\mathcal{F} k^2} + \frac{e^{-2\sqrt{\mathcal{F}} b k y}}{2\sqrt{\mathcal{F}} k^2 W(t)} \left[\left(\frac{\nu^2(t) + \alpha}{2\sqrt{\mathcal{F}} k} \right)^2 - \Omega(t) \right], \quad (3.19)$$

where $\mathcal{F} = (\hat{\mathcal{G}}^{\mathcal{P}} - k^2)/k^2$ and $W(t)$ is an arbitrary function that arises from the integration. Here we define the functions on the brane at $y = 0$ as $a_0(t) \equiv a(t, 0)$ and $\dot{a}_0(t) \equiv \dot{a}(t, 0) =$

$\nu(t)$, and fix the gauge $n(t, 0) = 1$. Then, eq. (3.19) on the brane at $y = 0$ can be written by

$$W(t) = \sqrt{\mathcal{F}} a_0^2 \left[1 + \frac{\dot{a}_0^2 + \alpha}{2\mathcal{F}k^2 a_0^2} \pm \left(1 + \frac{1}{\mathcal{F}k^2} \left(\frac{\dot{a}_0^2 + \alpha}{a_0^2} + \frac{\Omega(t)}{a_0^4} \right) \right)^{1/2} \right]. \quad (3.20)$$

We, therefore, obtain the exact solution of the scale factor expressed in terms of the brane scale factor, a_0 ,

$$a^2(t, y) = a_0^2 \left[\left(1 + \frac{\dot{a}_0^2 + \alpha}{2\mathcal{F}k^2 a_0^2} \right) \cosh(2\sqrt{\mathcal{F}}kby) \pm \left(1 + \frac{1}{\mathcal{F}k^2} \left(\frac{\dot{a}_0^2 + \alpha}{a_0^2} + \frac{\Omega(t)}{a_0^4} \right) \right)^{1/2} \sinh(2\sqrt{\mathcal{F}}kby) - \frac{\dot{a}_0^2 + \alpha}{2\mathcal{F}a_0^2 k^2} \right]. \quad (3.21)$$

From eqs. (3.1) and (3.21), the Friedmann equation on the brane is obtained by

$$H_0^2 + \frac{\alpha}{a_0^2} = k^2 - \frac{1}{6}(\mathcal{P}(X) - 2X\partial_X \mathcal{P}(X)) + \frac{\rho_0^2}{36M_{(5)}^8} - \frac{\Omega_0}{a_0^4}, \quad (3.22)$$

where $H_0 = \dot{a}_0/a_0$ is a Hubble's parameter defined on the brane and Ω_0 is the value of Ω at $y = 0$. Since we have eq. (3.6) on the brane at $y = 0$, eq. (3.22) can be rewritten by

$$H_0^2 + \frac{\alpha}{a_0^2} = -k^2 + \frac{1}{6}\mathcal{P}(X) + \frac{\rho_0^2}{36M_{(5)}^8} - \frac{\Omega_0}{a_0^4}. \quad (3.23)$$

All terms on the right-hand-side except the second term are familiar terms from the brane-world cosmological scenario.

The acceleration equation on the brane $y = 0$ can be obtained by combining the (44)-component of eq. (2.12) and eqs. (3.1), (3.2), (3.15), and (3.23),

$$\frac{\ddot{a}_0}{a_0} = 3k^2 - \frac{1}{2}\mathcal{P}(X) - \frac{\rho_0}{36M_{(5)}^8}(2\rho_0 + 3p_0) + \frac{\Omega_0}{a_0^4}. \quad (3.24)$$

In addition, it is easy to show that eqs. (3.23) and (3.24) satisfy the matter conservation equation on the brane,

$$\dot{\rho}_0 = -3H_0(\rho_0 + p_0). \quad (3.25)$$

Since we have $\mathcal{P}(X) = 6k^2 + CX$ in eq. (3.7), eq. (3.23) shows that the usual Friedmann equation can be described by taking C to be a positive constant (ordinary scalar field). Eqs. (3.23) and (3.24) indicate that the contribution of bulk matter to both equations can be described as a form of generalized bulk matter $\mathcal{P}(X)$, which is associated with the generalized *comoving mass* of the bulk fluid [20].

3.3 Consistent Conservation Equation

We can obtain a conservation law for X from eqs. (3.8) and (3.9) for consistent equations of motion. First, multiplying both sides of eq. (3.8) by a_0^2 , differentiating them with t , dividing by $2\dot{a}_0 a_0$, we obtain

$$\frac{\ddot{a}_0}{a_0} = \frac{1}{3}CX + \frac{\rho_0^2}{36M_{(5)}^8} + \frac{C\dot{X}}{6H_0} + \frac{\rho_0\dot{\rho}_0}{36M_{(5)}^8 H_0}. \quad (3.26)$$

This obviously should be equivalent to eq. (3.9), which yields the relation,

$$C \left(X + \frac{\dot{X}}{6H_0} \right) = -\frac{\rho_0}{36M_{(5)}^8 H_0} (\dot{\rho}_0 + 3H_0(\rho_0 + p_0)). \quad (3.27)$$

Since the RHS vanishes due to the conservation equation between brane tension and pressure, (3.25), we have a conservation equation for X

$$\dot{X} = -6H_0 X. \quad (3.28)$$

It is easy to show Eq. (3.28) has the solution $X = X_0/a_0^6$, where X_0 is an integration constant. Alternatively we can simplify eq. (3.15) when $y = 0$, obtaining $\Omega_0 = -CX_0/6a_0^2$ (assuming $\tilde{\Omega}(0) = 0$). Insertion of this into eqs. (3.23) and (3.24) produces

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{CX_0}{3a_0^6} + \frac{\rho_0^2}{36M_{(5)}^8}, \quad (3.29)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{2CX_0}{3a_0^6} - \frac{\rho_0}{36M_{(5)}^8} (2\rho_0 + 3p_0). \quad (3.30)$$

Note that these equations are coincident with eqs. (3.8) and (3.9). Since the X field does not couple to the brane tension and pressure, the conservation equations between X and ρ_0 (or/and p_0) are decoupled as shown before in eqs. (3.25) and (3.28).

Assuming $p_0 = \omega_b \rho_0$, where ω_b is an equation-of-state parameter of the 3-brane, eq. (3.25) has the solution

$$\rho_0 = \bar{\rho}_0 a_0^{-3(1+\omega_b)}, \quad (3.31)$$

where $\bar{\rho}_0$ is an integration constant whose value is negative, since the brane located at $y = 0$ is a negative tension brane in the RS1 model [1]. Therefore, we finally get the Friedmann and the acceleration equations on the brane in terms of the scale factor $a_0(t)$ for the static extra dimension

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{CX_0}{3a_0^6} + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}}, \quad (3.32)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{2CX_0}{3a_0^6} - \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} (2 + 3\omega_b). \quad (3.33)$$

We see from the above that for $C < 0$, we obtain acceleration in the brane. Unfortunately we cannot obtaining a condensing ghost field as eq. (3.7) shows. In the next section we address this problem.

4 Ghost Condensation From Radion Perturbation?

The simple ghost and scalar field solutions obtained in section 3 by assuming a static extra dimension do not lead to ghost condensation in the brane-world. In this section consider the possibility that a small perturbation of the radion (i.e. a non-static radius of the extra dimension) can give rise to ghost condensation. A ghost condensate can be regarded as a new sort of fluid that fills the universe [13]. Its essential property is that it has a non-vanishing time-dependent vacuum expectation value(VEV). Even though $\mathcal{P}(X)$ has a wrong-signed kinetic term, there exists a stable vacuum state with $\langle \phi \rangle \neq 0$. More precisely, the field equation of $\mathcal{P}(X)$ can have a solution, $\phi = ct$, where c is a dimensionless constant, provided that there exists a solution, $\partial_t \phi = \text{constant}$. By a small perturbation about this solution, $\phi = ct + \pi$, the lagrangian for quadratic fluctuations as shown in ref. [13] is given by

$$\mathcal{L} = M^4 \left[\left(\partial_X \mathcal{P}(c^2) + 2c^2 \partial_X^2 \mathcal{P}(c^2) \right) \dot{\pi}^2 - \partial_X \mathcal{P}(c^2) (\nabla \pi)^2 \right], \quad (4.1)$$

where we have discarded the term $2c\dot{\pi}\partial_X \mathcal{P}(c^2)$ since it is a total derivative. The usual signs can be recovered when c is such that

$$\partial_X \mathcal{P}(c^2) > 0, \quad \partial_X \mathcal{P}(c^2) + 2c^2 \partial_X^2 \mathcal{P}(c^2) > 0. \quad (4.2)$$

The typical shape and region we are considering are shown in Fig.1.

Let us consider a small radion perturbation $b = r_c + r_0(t)$, where r_c is a constant and $r_0 \ll r_c$. Using the metric (2.15), Einstein's equations on the brane at $y = 0$ can rewritten as

$$H_0^2 + H_0 \frac{\dot{r}_0}{r_c} + \frac{\alpha}{a_0^2} = \frac{1}{3} [2X \partial_X \mathcal{P}(X) - \mathcal{P}(X)] + 2k^2 + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}}, \quad (4.3)$$

$$\begin{aligned} & H_0^2 + 2 \left(\frac{\ddot{a}_0}{a_0} + H_0 \frac{\dot{r}_0}{r_c} \right) + \frac{\alpha}{a_0^2} \\ = & -\frac{\ddot{r}_0}{r_c} + 6k^2 - \mathcal{P}(X) + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} - \frac{2\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} (2 + 3\omega_b), \end{aligned} \quad (4.4)$$

$$H_0^2 + \frac{\ddot{a}_0}{a_0} + \frac{\alpha}{a_0^2} = 2k^2 - \frac{1}{3} \mathcal{P}(X) + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} - \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} (2 + 3\omega_b), \quad (4.5)$$

where we work to leading order in r_0 and set $\sigma(0) = 0$ (or $n_0 = n(t, 0) = 1$) on the brane. Combining these equations yields the following constraint between $\mathcal{P}(X)$ and the small perturbation of the radion field,

$$\frac{2}{3}(X\partial_X\mathcal{P}(X) - \mathcal{P}(X) + 6k^2) = 3H_0\frac{\dot{r}_0}{r_c} + \frac{\ddot{r}_0}{r_c}. \quad (4.6)$$

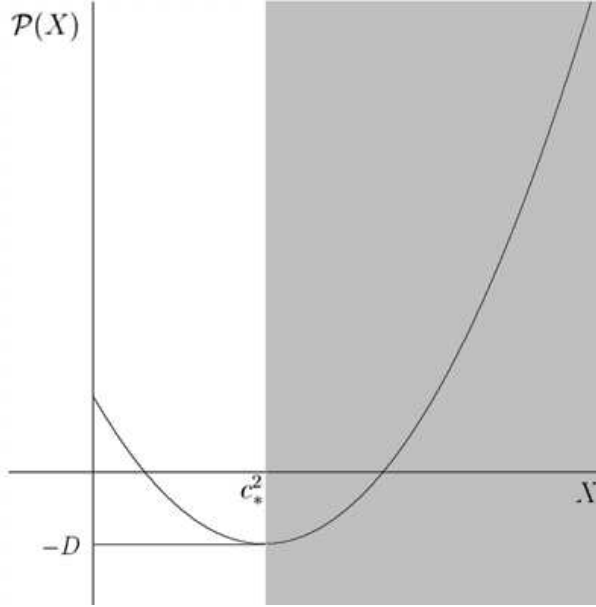


Figure 1: *The typical shape of $\mathcal{P}(X)$. A stability analysis [15] indicates that the shaded region is the one in which the ghost field behaves like an ordinary matter field and is stable, whereas the unshaded region is classically or quantum mechanically unstable.*

If we assume that the condensing ghost field approaches a stable vacuum at late times, then $\partial_X \mathcal{P}(c_*^2) \rightarrow 0$ and $\mathcal{P}(c_*^2) \rightarrow -D$ as shown in Fig.1. Defining $6k_0^2 = D + 6k^2$, Einstein's equations become in this limit

$$H_0^2 + \frac{\alpha}{a_0^2} = -H_0\frac{\dot{r}_0}{r_c} + 2k_0^2 + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}}, \quad (4.7)$$

$$\begin{aligned} H_0^2 + \frac{\alpha}{a_0^2} \\ = -2\left(\frac{\ddot{a}_0}{a_0} + H_0\frac{\dot{r}_0}{r_c}\right) - \frac{\ddot{r}_0}{r_c} + 6k_0^2 + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} - \frac{2\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}}(2 + 3\omega_b), \end{aligned} \quad (4.8)$$

$$H_0^2 + \frac{\alpha}{a_0^2} = -\frac{\ddot{a}_0}{a_0} + 2k_0^2 + \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}} - \frac{\bar{\rho}_0^2}{36M_{(5)}^8 a_0^{6(1+\omega_b)}}(2 + 3\omega_b), \quad (4.9)$$

and combining above three equations leads to two options: either $\dot{r}_0 = 0$ or $\ddot{a}_0/\dot{a}_0 = 0$.

For $\ddot{a}_0/\dot{a}_0 = 0$, we find that $a_0(t) = \mathcal{A}_1 t + \mathcal{A}_2$ where \mathcal{A}_1 and \mathcal{A}_2 are constants, which determines the solution for $r_0(t)$ but yields two options for making this so: $\omega_b = -1, -1/3$, depending upon the brane tension. These two choices of $\omega_b = -1$ and $\omega_b = -1/3$ produce the trivial equation $H_0^2 + \alpha/a_0^2 = 0$. Hence setting $\ddot{a}_0/\dot{a}_0 = 0$ does not yield any dynamical evolution of the scale factor since the effective energy density vanishes.

On the other hand, for $\dot{r}_0 = 0$, we obtain $k_0^2 = 0$ or $D = -6k^2$, which describes a vanishing cosmological constant at late times. The intriguing point is that the initially arbitrary value of D is determined by the consistency of Einstein's equations, leading to a vanishing cosmological constant ($k_0^2 = 0$) on the brane at late times. In this case, eq. (4.7) has solutions of the form for $\alpha = 0$,

$$a_0(t) = \mathcal{C}_1 e^{\frac{\bar{\rho}_0}{6M_{(5)}^4}} + \mathcal{C}_2 e^{-\frac{\bar{\rho}_0}{6M_{(5)}^4} t}, \quad (\omega_b = -1) \quad (4.10)$$

$$a_0(t) = 2^{-\frac{1}{3(1+\omega_b)}} \left[\text{const} \pm \frac{(1+\omega_b)\bar{\rho}_0}{M_{(5)}^4} t \right]^{\frac{1}{3(1+\omega_b)}}, \quad (\omega_b \neq -1)$$

$$\sim t^{\frac{1}{3(1+\omega_b)}}, \quad (4.11)$$

where \mathcal{C}_1 and \mathcal{C}_2 are integration constants.

Eq. (4.10) describes an inflationary solution for the minus (plus) sign since $\bar{\rho}_0 < 0$ ($\bar{\rho}_0 > 0$) for the RS1 (RS2) model. Although a detailed analysis of these solutions leads to a non-conventional cosmology [3], the usual FRW universe can be reproduced by various methods [4, 5, 6, 7, 8]. The crucial difference between this and the previous brane-world model is that the condensing ghost at late times cancels out the cosmological constant on the brane, which can lead to the usual brane-world cosmology with a vanishing cosmological constant.

Now consider the contribution of $\mathcal{P}(X)$ terms. Since this arbitrary kinetic function has a scale of $M_{(5)}^{-4}$ while the scale of the brane tension (pressure) terms is $M_{(5)}^{-8}$, we shall neglect the latter contribution. Furthermore, for a slowly varying extra dimension, we can neglect terms proportional to \ddot{r}_0 in eqs. (4.4) and (4.6). The equations of motion then simplify to

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{4}{9} X \partial_X \mathcal{P}(X) - \frac{1}{9} (\mathcal{P}(X) - 6k^2), \quad (4.12)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{4}{9} X \partial_X \mathcal{P}(X) - \frac{2}{9} (\mathcal{P}(X) - 6k^2), \quad (4.13)$$

$$\frac{2}{3} (X \partial_X \mathcal{P}(X) - \mathcal{P}(X) + 6k^2) = 3H_0 \frac{\dot{r}_0}{r_c}. \quad (4.14)$$

The typical shape of the ghost condensate is shown in Fig. 1. We shall take $\mathcal{P}(X)$ to have

the form

$$\mathcal{P}(X) = \frac{1}{2}(X - c_*^2)^2 + 6k^2, \quad (4.15)$$

which should be approximately true near $X = c_*^2$ (i.e. in the vicinity of the vacuum). With eq. (4.15), the equations of motion near the vacuum of the ghost condensate are

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{1}{18}(X - c_*^2)(7X + c_*^2) \equiv \frac{8\pi}{3}\rho_{gc}, \quad (4.16)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{1}{9}(X - c_*^2)(5X - c_*^2) \equiv -\frac{4\pi}{3}(\rho_{gc} + 3p_{gc}), \quad (4.17)$$

$$H_0 \frac{\dot{r}_0}{r_c} = \frac{1}{9}(X - c_*^2)(X + c_*^2), \quad (4.18)$$

where ρ_{gs} and p_{gs} represent respectively the energy density and the pressure generated by the ghost condensate. The first two equations describe the Friedmann and the acceleration equations while the third one is the evolution equation of the extra dimension. Note that the first two equations are similar to those shown in ref. [16] apart from some factors. The fact that eqs. (4.16) and (4.17) should satisfy the conservation equation, $\dot{\rho}_{gc} = -3H_0(\rho_{gc} + p_{gc})$, produces a differential equation for $X(t)$

$$\dot{X} = \pm \frac{(X - c_*^2)(17X - c_*^2)}{7X - 3c_*^2} \sqrt{\frac{(X - c_*^2)(7X + c_*^2)}{18}}, \quad (4.19)$$

and this equation is similar to that in ref. [16].

For small perturbations, the radion should be stable when X goes to the condensing vacuum, which implies that $r_0 < r_c$ should be valid for the solution of $r_0(t)$. Eqs. (4.18) and (4.19) becomes

$$\frac{dr_0}{dX} = -\frac{2r_c(7X - 3c_*^2)(X + c_*^2)}{(17X - c_*^2)(X - c_*^2)(7X + c_*^2)}, \quad (4.20)$$

which easily solves to

$$r_0(X) = \bar{r}_0 - r_c \left(\frac{33}{136} \ln(17X - c_*^2) + \frac{1}{8} \ln(X - c_*^2) - \frac{1}{4} \ln(7X + c_*^2) \right), \quad (4.21)$$

where \bar{r}_0 is an integration constant. The solution (4.21) shows that the radius of the extra dimension is destabilized when X goes to the stable vacuum, $X = c_*^2$, violating our assumption of small radion perturbations.

As a consequence, Minkowski spacetime at late times is not compatible with a stable radion field. To resolve this, we shall in the next section turn on the bulk matter along the y -direction, p_5 , and show that this stabilizes the radion field and alters the evolution of the scale factor and ghost field.

5 Stable Radion and Cosmic Acceleration

In this section, we consider the full-set of equations of motion with bulk matter along the y -direction, p_5 . We show that this can stabilize the radius of extra dimension and lead to the exponentially inflating scale factor and an accelerating expansion at late times. Introducing this sort of bulk matter has already shown in refs. [7, 17], which turns out to be obviously responsible for the radion stabilization.

5.1 Late-Time Behaviors by the Bulk Matter

We have three equations of motion for the slightly perturbed radius of extra dimension with the bulk matter, p_5 ,

$$H_0^2 + \frac{\alpha}{a_0^2} = -H_0 \frac{\dot{r}_0}{r_c} + 2k^2 + \frac{1}{3}(2X\partial_X\mathcal{P}(X) - \mathcal{P}(X)), \quad (5.1)$$

$$H_0^2 + \frac{\alpha}{a_0^2} + 2\left(\frac{\ddot{a}_0}{a_0} + H_0 \frac{\dot{r}_0}{r_c}\right) = 6k^2 - \mathcal{P}(X), \quad (5.2)$$

$$H_0^2 + \frac{\alpha}{a_0^2} + \frac{\ddot{a}_0}{a_0} = 2k^2 - \frac{1}{3}\mathcal{P}(X) - \frac{1}{3}p_5, \quad (5.3)$$

which are reduced to alternative form of equations by

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{4}{9}X\partial_X\mathcal{P}(X) - \frac{1}{9}\mathcal{P}(X) + \frac{2}{3}k^2 - \frac{2}{9}p_5, \quad (5.4)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{4}{9}X\partial_X\mathcal{P}(X) - \frac{2}{9}\mathcal{P}(X) + \frac{4}{3}k^2 - \frac{1}{9}p_5, \quad (5.5)$$

$$H_0 \frac{\dot{r}_0}{r_c} = \frac{2}{9}(X\partial_X\mathcal{P}(X) - \mathcal{P}(X) + 6k^2 + p_5). \quad (5.6)$$

On the other hand, the addition of p_5 should satisfies a condition for stabilization [17] given by $T^\mu_\mu - 2T^y_y = 0$, which leads to a general relation,

$$\mathcal{P}(X) - X\partial_X\mathcal{P}(X) = 6k^2 + p_5. \quad (5.7)$$

Assuming that $\partial_X\mathcal{P} \rightarrow 0$ and $\mathcal{P} \rightarrow -D$ as $t \rightarrow \infty$, where D is a constant from the condensing ghost at $X = c_*^2$, the set of equations becomes

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{2}{3}k_0^2 - \frac{2}{9}p_5, \quad (5.8)$$

$$\frac{\ddot{a}_0}{a_0} = \frac{4}{3}k_0^2 - \frac{1}{9}p_5, \quad (5.9)$$

$$H_0 \frac{\dot{r}_0}{r_c} = \frac{2}{9}(6k_0^2 + p_5). \quad (5.10)$$

The stabilization condition (5.7) becomes $p_5 + 6k_0^2 = 0$, which leads to a simple result of the stable radion, $r_0 = \text{const.}$, from eq. (5.10). And we have an inflationary solution of the scale factor for a flat ($\alpha = 0$) and the expanding universe ($k_0 > 0$),

$$a_0(t) = \bar{a}_0 e^{\sqrt{2}k_0 t}, \quad (5.11)$$

where \bar{a}_0 is an integration constant. Note that $6k^2 = \Lambda_{(5)}/M_{(5)}^4$ is negative (anti-de Sitter(AdS)), but the minimum value of ghost condensate ($D > 0$) renders the effective cosmological constant ($k_0^2 = D + 6k^2$) positive (dS phase), which ultimately can describe an accelerating universe at late times. In other words, an initial AdS spacetime becomes a dS spacetime at late times through the ghost condensation mechanism. We will show in the next section that ghost evolution also gives rise to this effect. As seen before, the bulk field, p_5 , stabilizes the radion at late times and alters the evolution of the scale factor to be inflationary.

5.2 Ghost Evolution and Accelerating Expansion of the Universe

For the near vacuum of the condensing ghosts, $\mathcal{P}(X) = (X - c_*^2)/2 - D$, eqs. (5.4), (5.5), and (5.6) can be written by

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{1}{18}(X - c_*^2)(7X + c_*^2) - \frac{2}{9}p_5 + \frac{2}{3}k_0^2, \quad (5.12)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{1}{9}(X - c_*^2)(5X - c_*^2) - \frac{1}{9}p_5 + \frac{4}{3}k_0^2, \quad (5.13)$$

$$H_0 \frac{\dot{r}_0}{r_c} = \frac{1}{9}(X - c_*^2)(X + c_*^2) + \frac{2}{9}p_5 + \frac{4}{3}k_0^2, \quad (5.14)$$

and eq. (5.7) can be evaluated by

$$p_5 = -6k_0^2 - \frac{1}{2}(X - c_*^2)(X + c_*^2), \quad (5.15)$$

which automatically leads to $r_0 = \text{const.}$ from eq. (5.14). It is easy to show that this result coincides with the one from the energy-momentum conservation, $\nabla_M T^{MN} = \nabla_M (T_{\mathcal{P}}^{MN} + T_{\text{bulk}}^{MN} + T_{\Lambda_{(5)}}^{MN}) = 0$. The field equation for X is equivalent to $\nabla_M T_{\mathcal{P}}^{MN} = 0$ and $\nabla_M (T_{\text{bulk}}^{Mt} + T_{\Lambda_{(5)}}^{Mt}) = 0$ yields

$$\frac{\dot{b}}{bn^2} p_5 = 0, \quad (5.16)$$

with the metric (2.2). Here if we set $b = r_c + r_0(t)$ for the small perturbation, we have $\dot{r}_0 = 0$ for the non-vanishing p_5 . We therefore obtain two equations using eq. (5.15),

$$H_0^2 + \frac{\alpha}{a_0^2} = \frac{1}{6}(X - c_*^2)(3X + c_*^2) + 2k_0^2, \quad (5.17)$$

$$\frac{\ddot{a}_0}{a_0} = -\frac{1}{6}(X - c_*^2)(3X - c_*^2) + 2k_0^2. \quad (5.18)$$

These equations are the same as those in ref. [16], and the analysis is similar. A plot of the deceleration parameter, $q_0 = -\ddot{a}_0/a_0 H_0^2$, and the equation-of-state parameter, ω appear in fig.2.

The only drawback to this model is the rather ad-hoc appearance of p_5 . One might hope that it could arise from the ghost field in some manner. The only possibility would appear to be giving the ghost field a y -dependence. However it is clear that this proposal cannot succeed. If we assume that $\phi = \phi(t, y)$, the stabilization condition becomes

$$\begin{aligned} & (2M_{(5)})^{-1} (T^\mu_\mu - 2T^y_y) \\ &= \mathcal{P}(X) - X\partial_X\mathcal{P}(X) - 3\partial_X\mathcal{P}(X)\frac{(\phi')^2}{b^2} - 6k^2 = 0. \end{aligned} \quad (5.19)$$

Comparing this with eq. (5.7), we have $p_5 = 3\partial_X\mathcal{P}(X)(\phi')^2/b^2$. However, this inevitably vanishes as t goes to infinity since $\partial_X\mathcal{P}(c_*^2) \rightarrow 0$ at late times, which finally leads to a Minkowski vacuum after condensation, yielding again an unstable radion field. Consequently the bulk field p_5 cannot originate from the ghost. However we note that it can arise from the back reaction of the dilaton coupling to the brane [17].

To summarize, introducing bulk matter stabilizes the radius of the extra dimension, alters the evolution of the ghost field and the scale factor, and prevents the cosmological constant from vanishing, which renders a dS phase on the brane and finally leads to an accelerating expansion at late times.

6 Higher-Derivative terms

Higher derivative terms for the ghost, in which $\mathcal{P}(X) = (X - c_*^2)^p + D$, where $p = 1, 2, 3, \dots$ can be analyzed in a similar manner. If $p_5 = 0$, we find that the situation can be simply classified by two cases : one for $p = \text{even}$ and another for $p = \text{odd}$. For $p = \text{even}$, the general behavior is quite similar to the preceding case for $p = 2$ up to the factor $1/2$, which results from the fact that there always exists a stable vacuum point at $X = c_*^2$. However, the odd power cases do not include the stable point at $X = c_*^2$, and so the situation is of no interest in connection with the issue of the ghost condensation. Nevertheless, one might consider two ways of circumventing the situation for $p = \text{odd}$. One is to regard the ghost condensate vacuum $X = c_*^2$ as a metastable state. Provided we can sit long enough at a point of inflection of $\mathcal{P}(X)$, these solutions could be viable for cosmological evolution, with the true condensate vacuum occurring the global minimum of $\mathcal{P}(X)$. Another way is

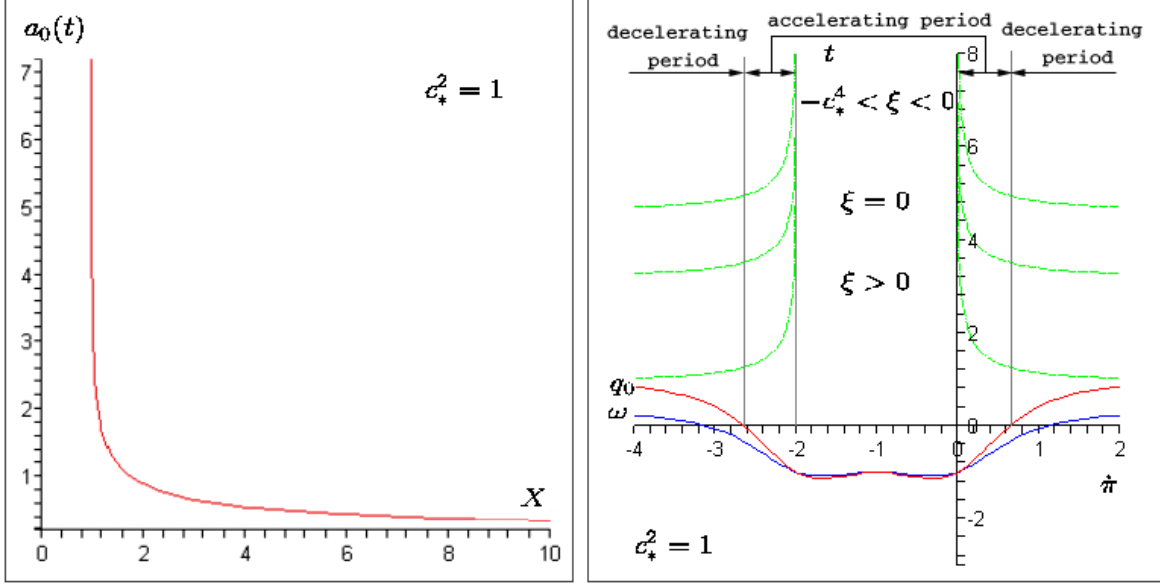


Figure 2: The behavior of the scale factor, a_0 , vs. X (LHS) and the plot of the solution for π (green dash-dot line), the deceleration parameter (red solid line), and the equation-of-state parameter (blue solid line) for each $\xi = 12k_0^2 - c_*^4$, where $\xi > -c_*^4$ (RHS)

to place the condensate minimum on the brane, writing $\mathcal{P}(X) = |X - c_*^2|^{2\gamma-1} + D$ where $\gamma = 1, 2, 3, \dots$. In this case, the discontinuity at $X = c_*^2$ is identified with the location of the brane at $y = 0$. Since we also have a discontinuity at $y = 0$ we could construct a new model for $p = \text{odd}$ power of ghosts that possesses a stable minimum.

As before, this situation can be addressed by introducing non-vanishing bulk matter, p_5 , with no additional modifications of the model. The bulk matter makes the radion stable and alters the evolution of the ghost condensate, whose equations reduce to

$$\frac{dX}{dt} = \pm \left[\frac{2X(X - c_*^2)\sqrt{3(X - c_*^2)^{p-1}((2p-1)X + c_*^2) + 18k_0^2}}{(2p-1)X - c_*^2} \right] \quad (6.1)$$

or alternatively

$$t - t_0 = \int dX \frac{(2p-1)X - c_*^2}{2X(X - c_*^2)\sqrt{3(X - c_*^2)^{p-1}((2p-1)X + c_*^2) + 18k_0^2}} \quad (6.2)$$

which have not been previously analyzed. It is straightforward to show that the integrands of eq. (6.2) for each $p \neq 1$ have qualitatively similar behavior. Consequently the evolution of higher derivative ghosts with bulk matter is qualitatively equivalent to the $p = 2$ case for both p even and odd.

We can obtain some analytic information about the evolution of the condensate in the large t limit. Since X goes to c_*^2 we can employ the ansatz $X = c_*^2 + f(t)e^{-\mu t}$, where μ is a positive constant. This induces a differential equation for $f(t)$ which is

$$\frac{df}{dt} = \left(\mu f \pm \frac{2\sqrt{3}f(c_*^2 + f e^{-\mu t})\sqrt{(f e^{-\mu t})^{p-1}(2pc_*^2 + (2p-1)f e^{-\mu t}) + 6k_0^2}}{2c_*^2(p-1) + (2p-1)f e^{-\mu t}} \right). \quad (6.3)$$

For large t , $\exp(-\mu t)$ goes to zero, which reduces the differential equation to

$$\frac{df}{dt} = \left[\left(\mu \pm \frac{3\sqrt{2}k_0}{p-1} \right) f \mp \frac{3\sqrt{2}k_0 f^2}{2(p-1)^2 c_*^2} e^{-\mu t} \right]. \quad (6.4)$$

To solve this, we note that f should be a constant (i.e. $df/dt = 0$) at large t . This determines $\mu = \pm 3\sqrt{2}k_0/(p-1)$, where we take the plus sign to ensure a decaying solution for X .

Now we consider the next leading term of eq. (6.4). Plugging the value of μ into eq. (6.4) leads to the solution,

$$f(t) = \left(A + \frac{e^{-\frac{3\sqrt{2}k_0}{p-1}t}}{2c_*^2(p-1)} \right)^{-1} \quad (6.5)$$

where $A > 0$ is an integration constant. Eq. (6.5) shows that the function $f(t)$ becomes a constant as t goes to infinity and the solution of X is

$$X = c_*^2 + e^{-\frac{3\sqrt{2}k_0}{p-1}t} \left(A + \frac{e^{-\frac{3\sqrt{2}k_0}{p-1}t}}{2c_*^2(p-1)} \right)^{-1}, \quad (6.6)$$

illustrating that the solutions have qualitatively similar behavior. Since we are working in the near-vacuum region, the effects of higher order terms beyond the leading contribution at a given p will be negligible at late times.

7 Conclusions

We have studied the RS brane-world model with an arbitrary function $\mathcal{P}(X)$ of the kinetic term of a scalar field in the five-dimensional AdS spacetimes. An interesting aspect of our model is that the generic function $\mathcal{P}(X)$ is uniquely determined in the brane-world with a static extra dimension. Once time-evolution of the radius of the extra dimension is taken into account, the solution gives rise to ghost condensation. This implies that the excitation of the brane along the extra coordinate results in a ghost condensate and the ghost field approaches the stable vacuum at late times. Another intriguing feature of this model is that the minimum value of the ghost condensate vacuum cancels out the cosmological constant.

This inevitably leads to the Minkowskian vacuum on the brane after the ghost condensation at late times. In this background, a radiation-dominated universe can be generated by the vacuum fluctuations of the ghost condensate. As a result, vacuum fluctuations of ghosts can generate radiating matter at late times by the ghost condensation.

However, this scenario inevitably yields an unstable radion. This instability can be circumvented by introducing a bulk field along the y -direction, preserving the consistency of the equations of motion and reducing the system to that of the four-dimensional case [16]. This sort of bulk matter along the extra dimension can arise from the back reaction of the dilaton field [17], which is obviously responsible for the stabilized radion. Since it alters the time-evolution of the scale factor and the ghost field, the model ultimately leads to the inflationary expanding scale factor and the acceleratingly expanding epoch at late times. In addition, the condensing ghost behaves like a cosmological constant since it approaches to $\omega = -1$ as times goes on. One of the interesting features in this model is that the spacetimes with an initially negative curvature (AdS) transfers to the spacetimes with a positive curvature (dS) as the condensing ghost approaches to the stable vacuum, which results from the fact that the vacuum of the ghost condensation (D) forbids the effective cosmological constant (k_0^2) to be negative or vanish.

For $p = 2$ an analysis of our model has already been carried out in ref. [16]. However there are some distinctions between the two models. The main point of our model was to investigate if/how ghost condensation can appear from a wiggling brane along the extra dimension. One of the interesting features of our model is that the zero point energy (D -term) of ghost condensation inevitably must appear in order to preserve consistent equations of motion, which are ultimately responsible for the dS expansion at late times. The situation for larger p has not been previously analyzed; we find that the behaviour of the ghost condensate is qualitatively similar to the $p = 2$ case.

As a consequence, ghost condensation in the RS brane-world model with the stabilized radius of the extra dimension provides a possible explanation of an accelerating universe at late times. In addition, the model might offer a possible account of an early universe inflationary cosmology, which would be worthwhile to explore in the future.

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